

CBCS SCHEME

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15MAT41

Fourth Semester B.E. Degree Examination, July/August 2021 Engineering Mathematics – IV

Time: 3 hrs.

Max. Marks:80

Note: Answer any FIVE full questions.

- 1 a. Find y at $x = 0.4$ correct to 4 decimal places given $\frac{dy}{dx} = 2xy + 1$; $y(0) = 0$ applying Taylor's series method upto third degree term. (05 Marks)
- b. Using modified Euler's method find $y(0.2)$ correct to four decimal places solving the equation $y' = x - y^2$, $y(0) = 1$ taking $h = 0.1$. Use modified Euler's formula twice. (05 Marks)
- c. Use fourth order Runge – Kutta method to solve $(x + y)\frac{dy}{dx} = 1$, $y(0.4) = 1$ at $x = 0.5$ correct to four decimal places. (06 Marks)

- 2 a. Using Runge-Kutta method of fourth order, find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ by taking $h = 0.2$. (05 Marks)
- b. Apply Milne's method to find y at $x = 1.4$ correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and the following data $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$. (05 Marks)
- c. Find the value of y at $x = 4.4$ by applying Adams – Bashforth method given that $5x\frac{dy}{dx} + y^2 - 2 = 0$ with the initial values of y : $y_0 = 1$, $y_1 = 1.0049$, $y_2 = 1.0097$, $y_3 = 1.0142$ corresponding to the values of x : $x_0 = 4$, $x_1 = 4.1$, $x_2 = 4.2$, $x_3 = 4.3$. (06 Marks)

- 3 a. Apply Milne's predictor – corrector method to compute $y(0.4)$ given the differential equation $y'' + 3xy' - 6y = 0$ and the following table of initial values. (05 Marks)

x	0	0.1	0.2	0.3
y	1	1.03995	1.13803	1.29865
y'	0.1	0.6955	1.258	1.873
- b. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x$. (05 Marks)
- c. Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomials. (06 Marks)

- 4 a. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$, compute $y(0.2)$ using fourth order Runge – Kutta method. (05 Marks)
- b. Prove the Rodrigues formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$. (05 Marks)
- c. Obtain the series solution of Bessel's differential equation $x^2 y'' + xy' + (x^2 + n^2)y = 0$. (06 Marks)

- 5 a. State and prove Cauchy's – Riemann equation in polar form. (05 Marks)
- b. Discuss the transformation $W = Z^2$. (05 Marks)
- c. Using Cauchy's residue theorem evaluate : (06 Marks)

$$\int_C \frac{z \cos z}{(z - \frac{\pi}{2})^3} dz \quad \text{where} \quad C : |z - 1| = 1.$$

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 6 a. Find an analytical function whose real part is $e^{-x}[(x^2 - y^2)\cos y + 2xy\sin y]$. (05 Marks)
- b. Evaluate: $\int_C \frac{e^{2z}}{(z+1)(z-2)} dz$ where C is the circle $|z| = 3$. (05 Marks)
- c. Find the bilinear transformation which maps the points $Z = 1, i, -1$ into $w = 0, 1, \infty$. (06 Marks)

- 7 a. A random variate X has the following probability function for various values of x

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K ²	2K ²	7K ² + K

- Find: i) K ii) Evaluate $P(x < 6)$ $P(x \geq 6)$ and $P(0 < x < 5)$. (05 Marks)
- b. Find the mean and standard deviation of the exponential distribution. (05 Marks)
- c. The joint probability distribution table for two random variables X and Y as follows:

Y \ X	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine:

- i) Marginal distribution of X and Y
 ii) Expectation of X
 iii) S.D of Y
 iv) Covariance of X and Y
 v) Correlation of X and Y. (06 Marks)
- 8 a. A random variable x has the following density function:

$$f(x) = \begin{cases} Kx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

- Evaluate: i) K ii) $P(1 < x < 2)$ iii) $P(x \leq 1)$ iv) $P(x > 1)$ v) Mean. (05 Marks)
- b. In a quiz contest of answering 'Yes' or 'No' what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer. (05 Marks)
- c. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and S.D of the distribution. It is given that if:

$$P(Z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2} dz$$

then $A(-0.4958) = 0.19$ and $A(1.405) = 0.42$. (06 Marks)

- 9 a. The weights of 1500 ball bearings are normally distributed with a mean of 635gms and S.D of 1.36gms. If 300 random samples of size 36 are drawn from this population, determine the expected mean and S.D of the sampling distribution of means if sampling is done :
i) with replacement ii) without replacement. (05 Marks)
- b. Two athletes A and B were tested according to the time (in seconds) to run a particular race with the following results.

Athlete A	28	30	32	33	33	29	34
Athlete B	29	30	30	24	27	29	

Test whether you can discriminate between the two Athletes. ($t_{0.05} = 2.2$ and $t_{0.02} = 2.72$ for 11d.f). (05 Marks)

- c. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study? (06 Marks)
- 10 a. The mean and S.D of the maximum loads supported by 60 cables are 11.09 tonnes and 0.73 tonnes respectively. Find : i) 95% ii) 99% confidence limits for mean of the maximum loads of all cables produced by the company. (05 Marks)
- b. Fit a Poisson distribution for the following data and test the goodness of fit given that $\chi^2_{0.05} = 7.815$ for 3d.f.

x	0	1	2	3	4
f	122	60	15	2	1

(05 Marks)

- c. Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is a regular stochastic matrix. Also find the associated unique fixed probability vector. (06 Marks)

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15EC43

Fourth Semester B.E. Degree Examination, July/August 2021 Control Systems

Time: 3 hrs.

Max. Marks:80

Note: Answer any FIVE full questions.

- 1 a. List the merits and demerits of open loop and closed loop control systems. Give at least one example each? (05 Marks)
- b. Explain the block diagram rule regarding : (05 Marks)
 - i) Combining blocks in cascade
 - ii) Moving a summing point after a block
 - iii) Moving a take-off point beyond a block.
- c. For the electrical circuit shown in Fig.Q1(c) construct the block diagram.

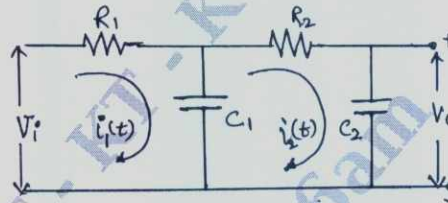


Fig.Q1(c)

(06 Marks)

- 2 a. For the two port network shown in Fig.Q2(a), obtain transfer function of $V_1(s)/I_1(s)$.



Fig.Q2(a)

(05 Marks)

- b. For the system shown in Fig.Q2(b), i) Write the differential equation describing the system
ii) Draw Force – voltage analogous electrical circuit.

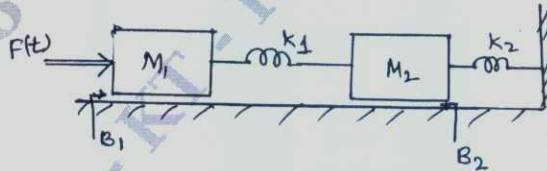


Fig.Q2(b)

(05 Marks)

- c. For the signal graph shown in Fig.Q2(c), find the transfer function, using Masoris gain formula.

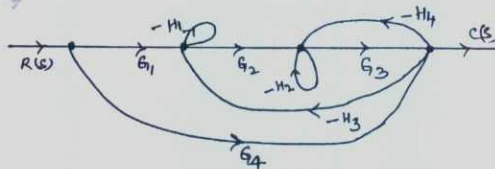


Fig.Q2(c)

(06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 3 a. Derive the time response of second order system for the underdamped case, subjected to unit step input. (10 Marks)
- b. For the unity feedback system $G(s) = \frac{s(s+1)}{s^2(s+3)(s+10)}$. Determine the type of the system, error co-efficients and steady state error for input $r(t) = 1 + 3t + \frac{t^2}{2}$. (06 Marks)
- 4 a. Explain the following time domain specifications of a second order system :
i) Rise time ii) Peak time iii) Maximum over shoot iv) Settling time. (04 Marks)
- b. The open loop transfer function of a unity feedback control system is given as $G(S) = \frac{k}{S(TS+1)}$. By what factor the system gain K has to be multiplied to decrease overshoot from 75% to 25%. (06 Marks)
- c. For the system shown in Fig.Q4(c), determine the value of 'a' which gives damping factor 0.7. What is the steady state error to unit ramp input for valve of 'a'.

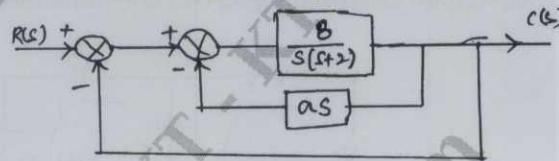


Fig.Q4(c)

- (06 Marks)
- 5 a. Explain the terms 'Relative stability' and 'Conditional Stability'. (04 Marks)
- b. State and explain Routh-Hurwitz criterion. (04 Marks)
- c. Sketch the root loci to determine the stability of the system $G(s) = \frac{k}{s(s+1)(s+3)}$. (08 Marks)
- 6 a. State the different rules for the construction of root locus. (08 Marks)
- b. The open loop transfer function of a unity feedback system is given by $G(s) = \frac{k}{s(s+3)(s^2+s+1)}$. Find the value of K, that will cause sustained oscillation and hence find the oscillation frequency. (08 Marks)
- 7 a. Define the following with reference to Bode plots :
i) Gain margin
ii) Phase margin
iii) Gain cross over frequency
iv) Phase cross over frequency. (06 Marks)
- b. Construct the bode plot for a unity feedback control system with $G(s) = \frac{10(s+10)}{s(s+2)(s+5)}$. Find its gain margin and phase margin. Comment on the stability. (10 Marks)
- 8 a. Sketch the polar plot of $G(s) = \frac{1}{s+2}$. Show all the steps involved. (06 Marks)
- b. Sketch the Nyquist plot for the open - loop transfer function $G(s)H(s) = \frac{10}{(s+2)(s+4)}$. Determine the stability of the closed loop system by Nyquist criterion. (10 Marks)

- 9 a. List the properties of state transition matrix. (06 Marks)
 b. What is sampled data control system? (02 Marks)
 c. Obtain the state model for the electric network shown in Fig.9(c). Select i_L and V_c as state variables.

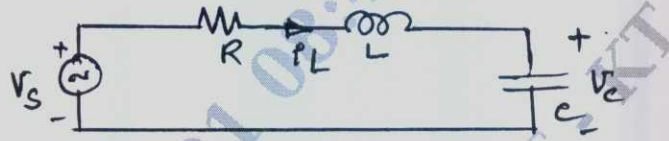


Fig.Q9(c)

- 10 a. What is signal reconstruction? Explain it with sample and hold circuit. (08 Marks)
 b. Obtain the state transition matrix $Q(t)$ of the following system. (08 Marks)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Also obtain the inverse of the state transition matrix $\phi'(t)$.

(08 Marks)

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USN

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15EC44

Fourth Semester B.E. Degree Examination, July/August 2021 Signals and Systems

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions.

- 1 a. Check whether the following system is i) linear or nonlinear ii) Time invariant or time variant iv) Stable or unstable v) invertible or non invertible. $y(n) = \log(x(n))$ (06 Marks)
- b. Sketch the following signals and determine their even and odd components $x(n) = u(n+2) - 3u(n-1) + 2u(n-5)$ (06 Marks)
- c. Represent the given signal $x(t)$ shown in Fig Q1(c) using basic signals. (04 Marks)

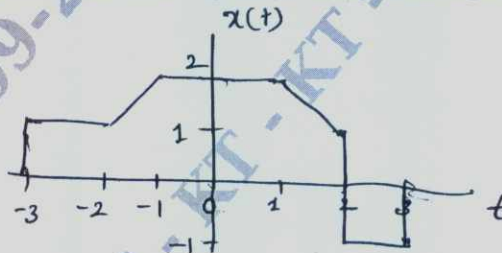


Fig Q1(c)

(04 Marks)

- 2 a. Check whether the following signal are periodic or not. If periodic, determine the fundamental period :
 - i) $x(n) = \cos\left(\frac{\pi n}{7}\right) \sin\left(\frac{\pi n}{3}\right)$
 - ii) $x(t) = \left[2\cos^2\left(\frac{\pi t}{2}\right) - 1\right] \cos(\pi t) \sin(\pi t)$ (06 Marks)
- b. A rectangular pulse $x(t) = \begin{cases} A, & 0 \leq t \leq T \\ 0, & \text{Elsewhere} \end{cases}$ is applied to an integrator circuit, find the total energy of the output $y(t)$ of the integrator. (05 Marks)
- c. A staircase signal $x(t)$ that may be viewed as a superposition of four rectangular pulses. Starting with rectangular pulse shown in Fig Q2(c), constant and express $x(t)$ in forms of $g(t)$

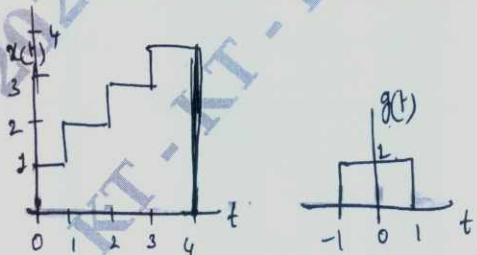


Fig Q2(c)

(05 Marks)

- 3 a. Find the overall impulse response of a cascade of two systems having identical impulse responses, $h(t) = 2[u(t) - u(t-1)]$. (08 Marks)
- b. Find the discrete time convolution sum given below. $y(n) = \beta^n u(n) * \alpha^n u(n)$; $|\beta| < 1$; $|\alpha| < 1$. (08 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

- 4 a. A LTI system has impulse response $h(t) = t u(t) + (10 - 2t) u(t - 5) - (10 - t) u(t - 10)$. Determine the output for the following input $x(t) = \delta(t + 2) + \delta(t - 5)$. (05 Marks)
- b. Evaluate the discrete time convolution sum given below $y(n) = u(n) \times u(n - 3)$. (08 Marks)
- c. State three properties of discrete time convolution. (03 Marks)
- 5 a. Find the step response of a system whose impulse response is given by $h(t) = u(t + 1) - u(t - 1)$. (04 Marks)
- b. A system consists of several subsystem connected as shown in Fig Q5(b). Find the operator H relating $x(t)$ and $y(t)$ for the following subsystem operators.
- $H_1 : y_1(t) = x_1(t) x_1(t - 1)$
 $H_2 : y_2(t) = |x_2(t)|$
 $H_3 : y_3(t) = 1 + 2x_3(t)$
 $H_4 : y_4(t) = \text{Cos}(y_3(t))$

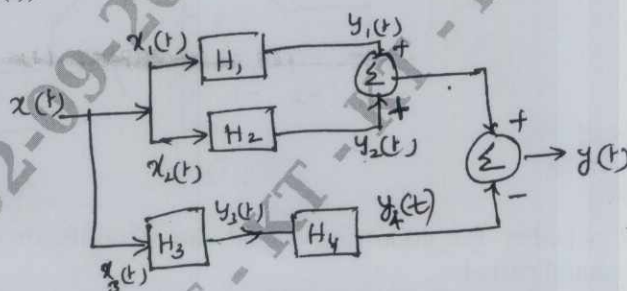


Fig Q5(b)

- (05 Marks)
- c. Obtain the DTFS coefficient of $x(n) = \text{Cos}\left(\frac{6\pi}{13}n + \frac{\pi}{6}\right)$.
 Draw : i) Magnitude spectrum ii) Phase spectrum. (07 Marks)
- 6 a. State the following properties of DTFS.
 i) Linearity ii) Time shift iii) frequency shift iv) Parseval's Relationship v) Convolution vi) Modulation. (06 Marks)
- b. Evaluate the FS representation for the signal, $x(t) = \text{Sin}(2\pi t) + \text{Cos}(3\pi t)$. Sketch the magnitude and phase spectra. (07 Marks)
- c. For the impulse response $h(n)$ given below determine whether the corresponding system is
 i) memoryless ii) causal iii) stable.
 $h(n) = 2u(n) - 2u(n - 1)$. (03 Marks)
- 7 a. Compute the DTFT of the signal
 $x(n) = \left(\frac{1}{2}\right)^n \{u(n+3) - u(n-2)\}$ (06 Marks)
- b. State and prove the following properties of Fourier Transform.
 i) Frequency differentiation ii) Linearity. (06 Marks)
- c. State Sampling theorem. (04 Marks)
- 8 a. Specify the Nyquist rate and Nyquist intervals for the following signals.
 i) $g(t) = \text{Sin} c(200t)$ ii) $g_2(t) = \text{Sinc}^2(200t)$. (04 Marks)
- b. Obtain the Fourier transform of the signal $x(t) = e^{-at} u(t)$; $a > 0$. Draw its magnitude and phase spectra. (06 Marks)
- c. State and explain the significance of following terms under DTFT
 i) Parseval's relation ii) Convolution iii) Time shift. (06 Marks)

- 9 a. Explain the properties of ROC. (04 Marks)
- b. Determine the z-transform of $x(n) = -u(-n-1) + \left(\frac{1}{2}\right)^n u(n)$. Find the ROC and pole-zero locations of $x(z)$ in the Z-plane. (06 Marks)
- c. A causal system has input $x(n]$ and output $y[n]$. Find the impulse response of the system, if
- $$x(n) = \delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{8}\delta(n-2)$$
- $$y(n) = \delta(n) - \frac{3}{4}\delta(n-1)$$
- (06 Marks)
- 10 a. State and prove the following properties Z-transform i) Initial value theorem ii) Time reversal property. (06 Marks)
- b. Find the inverse Z-transform of $x(z) = \frac{z^{-1}}{-2z^{-2} - z^{-1} + 1}$ ROC : $|z| < 2$. (06 Marks)
- c. Determine whether the system is causal and stable $H(z) = \frac{2z+1}{z^2+z-5/16}$. (04 Marks)

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15EC45

Fourth Semester B.E. Degree Examination, July/August 2021 Principle of Communication Systems

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions.

1. a. Explain in detail Quadrature carrier multiplexing and demultiplexing systems. (05 Marks)
b. With relevant equations and diagrams explain the generation of AM waves using switching modulator. (05 Marks)
c. Consider a two stage modulator where the message signal occupies a band of 0.3KHz to 4 kHz and the two carrier frequencies are $f_1 = 10\text{KHz}$ and $f_2 = 100\text{KHz}$. Evaluate the following :
 - i) Sidebands of DSB – SC waves at the output of product modulators
 - ii) Sideband at the output of Band pass filters
 - iii) Passbands and guard bands of two BPF's
 - iv) The order of the two filters assuming at least 15dB attenuation between the passband and stop band. (06 Marks)
2. a. Explain the working of practical synchronous cost as receiver system for demodulating DSB – SC wave. (06 Marks)
b. Define percentage modulation with relevant equation illustrate the time domain and frequency domain characteristics of single Tone amplitude modulated wave. (05 Marks)
c. An audio frequency signal $10\text{Sin } 2\pi \times 500t$ is used to amplitude modulate a carrier of $50 \text{ Sin } 2\pi \times 10^5 t$. Calculate i) Modulation index ii) Sideband frequencies iii) Amplitude of each sideband iv) Bandwidth v) Total power delivered to the load of 600Ω vi) Plot the frequency spectrum. (05 Marks)
3. a. Define modulation index, frequency deviation and derive the time domain and frequency domain representation of wide band FM. (07 Marks)
b. With relevant diagram, explain the balanced slope method of FM demodulation. (05 Marks)
c. An angle modulated signal is represented by $\delta(t) = 10 \text{ Cos}[2\pi \times 10^6 t + 5 \text{ Sin } 2000\pi t + 10 \text{ Sin } 3000 \pi t]$ volts. Find the following :
 - i) The power in the modulated signal across 1Ω resistor
 - ii) Frequency deviation
 - iii) The deviation ratio
 - iv) The phase deviation
 - v) The approximate transmission Bandwidth, B_T . (04 Marks)
4. a. With block diagram, explain the linear model of PLL. (08 Marks)
b. Write short notes on Non linearity and its effects in FM system. (04 Marks)
c. Explain FM stereo multiplexing in detail. (04 Marks)
5. a. For a random process $X(t)$, define mean, correlation and covariance function. Explain the properties of autocorrelation function. (06 Marks)
b. In a communication receiver, the first stage is a tuned amplifier with an available power gain of 20dB and a noise figure of 10dB. The output of the amplifier is given to the mixer stage whose noise figure is 20dB. Determine the overall noise figure of the system. (05 Marks)

- c. Show that the area under probability density function curve is always equal to unity. (05 Marks)
- 6 a. Define white noise and plot the power spectral Density and autocorrelation function of Ideal low pass filtered white noise. (08 Marks)
b. Define Noise equivalent Bandwidth and derive the expression for the same. (08 Marks)
- 7 a. Show that the figure of merit of a noisy FM receiver for single tone modulation is $\frac{3}{2}\beta^2$. (08 Marks)
b. With neat diagram, explain in detail the noisy receiver model. (05 Marks)
c. Explain the capture effect in FM. (03 Marks)
- 8 a. Derive the equation for the figure of merit of an AM receiver operating on a single tone AM. (06 Marks)
b. Explain FM threshold effect and its reduction methods. (04 Marks)
c. Give the importance of pre-emphasis and de-emphasis in frequency modulation. (06 Marks)
- 9 a. State and explain sampling theorem. State Nyquist rate and Nyquist interval. (10 Marks)
b. Calculate the nyquist rate and nyquist interval for
i) $x(t) = 3 \cos 50\pi t + 10 \sin 300\pi t + \cos 100\pi t$
ii) $x(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$ (06 Marks)
- 10 a. Explain Quantization process Quantization noise and show that the output signal to noise ratio of an uniform quantize increases exponentially with the increasing number of bit per sample. (10 Marks)
b. With neat block diagram, explain TDM system. (06 Marks)

CBCS SCHEME

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15MATDIP41

Fourth Semester B.E. Degree Examination, July/August 2021

Additional Mathematics – II

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions.

- 1 a. Determine the rank of the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by applying elementary row transformations. (05 Marks)
- b. Find the inverse of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ using Cayley Hamilton theorem. (05 Marks)
- c. Solve by Gauss elimination method
 $2x + y + 4z = 12$
 $4x + 11y - z = 33$
 $8x - 3y + 2z = 20$ (06 Marks)
- 2 a. Find the eigen values of $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ (05 Marks)
- b. Solve the system of equations by Gauss elimination method.
 $x + y + z = 9$
 $x - 2y + 3z = 8$
 $2x + y - z = 3$ (06 Marks)
- c. Find the rank of the matrix by reducing it to echelon form.
 $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ (05 Marks)
- 3 a. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$ subject to $\frac{dy}{dx} = 2, y = 1$ at $x = 0$. (05 Marks)
- b. Solve $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$. (05 Marks)
- c. Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + y = \tan x$. (06 Marks)
- 4 a. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \cos 2x$. (05 Marks)
- b. Solve $y'' + 2y' + y = 2x + x^2$ (05 Marks)
- c. Using the method of undetermined coefficients, solve $y'' - 5y' + 6y = e^{3x} + x$ (06 Marks)

- 5 a. Find the Laplace transform of (i) $\frac{e^{-at} - e^{-bt}}{t}$ (ii) $\sin 5t \cos 2t$ (05 Marks)
- b. Find the Laplace transform of $f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases}$ where $f(t+a) = f(t)$ (06 Marks)
- c. Express $f(t) = \begin{cases} t, & 0 < t < 4 \\ 5, & t > 4 \end{cases}$ in terms of unit step function and hence find $L[f(t)]$. (05 Marks)
- 6 a. Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. (06 Marks)
- b. Find the Laplace Transform of (i) $t \sin at$ (ii) $t^5 e^{4t}$ (05 Marks)
- c. If $f(t) = t^2$, $0 < t < 2$ and $f(t+2) = f(t)$ for $t > 2$, find $L[f(t)]$. (05 Marks)
- 7 a. Find the inverse Laplace Transform of $\frac{2s-1}{s^2+4s+29}$. (05 Marks)
- b. Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s}{a}\right)$. (05 Marks)
- c. Solve by using Laplace Transforms $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}$; $y(0) = 0$, $y'(0) = 0$. (06 Marks)
- 8 a. Solve the initial value problem $y'' + 4y' + 3y = e^{-t}$ conditions with $y(0) = 1$, $y'(0) = 1$ using Laplace Transforms. (06 Marks)
- b. Find the inverse Laplace Transform of $\frac{s+2}{s^2(s+3)}$ (05 Marks)
- c. Find the inverse Laplace Transform of $\log\left[\frac{s^2+4}{s(s+4)(s-4)}\right]$ (05 Marks)
- 9 a. A box contains 3 white, 5 black and 6 red balls. If a ball is drawn at random, what is the probability that it is either red or white? (05 Marks)
- b. The probability that a person A solves the problem is $\frac{1}{3}$, that of B is $\frac{1}{2}$ and that of C is $\frac{3}{5}$. If the problem is simultaneously assigned to all of them what is the probability that the problem is solved? (05 Marks)
- c. Three machines A, B and C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentages of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. (06 Marks)
- 10 a. State and prove Baye's theorem. (05 Marks)
- b. If A and B are events with $P(A \cup B) = \frac{3}{4}$, $P(\bar{A}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, find $P(A)$, $P(B)$ and $P(A \cap \bar{B})$. (05 Marks)
- c. Three students A, B, C, write an entrance examination. Their chances of passing are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability that (i) atleast one of them passes (ii) all of them pass (iii) atleast two of them passes. (06 Marks)
